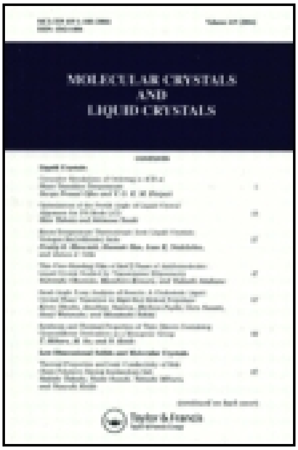

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Flow Induced Switching in a Bistable Nematic Device

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This paper discusses some recent experimental results concerning the switching of a weakly anchored nematic liquid crystal between bistable states by exploiting backflow in the cell.

Keywords: nematic; bistable device; backflow; weak anchoring; surface viscosity

INTRODUCTION

In a recent paper Dozov. Nobili and Durand[1] describe experiments in which they switch a weakly anchored nematic liquid crystal between two bistable states by applying voltages across the cell. The bistable states are both aligned parallel to the plates, being uniform in one but having a π-twist in the other, these configurations having equal energy if the nematic contains an appropriate amount of chiral dopant. Also the surface anchoring is weak and of unequal strengths at the two plates. Application of a voltage within a given range breaks the anchoring only at the surface with the weaker anchoring, and thereby achieves the transition from the twisted state to the uniform state. However, the reverse switching is more complex. A sufficiently strong voltage to break the anchoring at both plates is removed abruptly, and this leads to the nematic adopting a state involving bend at both plates, which ultimately relaxes to a π-twist. The initial stage of this relaxation into the splay-bend state is attributed to viscous coupling between the plates through the presence of backflow.

Our aim in this paper is to investigate the role of flow in the first stage of this latter relaxation employing continuum theory, albeit with a somewhat simplified model to reduce the numerical calculations involved. In this way, our investigation confirms the arguments put forward by Dozov, Nobili and Durand[1], by showing that the flow induced initially at the more strongly anchored surface can influence the alignment at the weaker surface to produce a bend–splay rotation of π across the cell.

CONTINUUM EQUATIONS

Below we employ the continuum equations proposed by Ericksen and Leslie (see, for example, Leslie[2] or de Gennes and Prost[3]), although in this preliminary study it seems reasonable to use a simplified model to reduce the numerical computations. Here we consider solutions of their equations in which the velocity $\underline{v}$ and alignment $\underline{n}$ take the forms referred to Cartesian axes

$$\underline{v} = (u(z,t), 0, 0), \quad \underline{n} = (\sin\theta(z,t), 0, \cos\theta(z,t)). \tag{1}$$

where z is the Cartesian coordinate normal to the plates, and t denotes time. In this event it follows that u and θ must satisfy

$$\rho\frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\left(g(\theta)\frac{\partial u}{\partial z} - m(\theta)\frac{\partial\theta}{\partial t}\right) \tag{2}$$

and

$$2f(\theta)\frac{\partial^2\theta}{\partial z^2} + \frac{df(\theta)}{d\theta}\left(\frac{\partial\theta}{\partial z}\right)^2 - 2\gamma_1\frac{\partial\theta}{\partial t} + 2m(\theta)\frac{\partial u}{\partial z} - 2\epsilon_a E^2\sin\theta\cos\theta = 0. \tag{3}$$

where

$$\begin{aligned} 2g(\theta) &= \alpha(1 + 2\cos^2\theta), \quad m(\theta) = \alpha\cos^2\theta, \\ f(\theta) &= K_1\sin^2\theta + K_3\cos^2\theta, \quad \gamma_1 = \alpha. \end{aligned} \tag{4}$$

having chosen

$$\alpha_1 = \alpha_3 = \alpha_6 = 0, \quad \alpha_4 = \alpha_5 = -\alpha_2 = \alpha. \tag{5}$$

Also for very thin layers it seems reasonable to neglect the fluid inertia by setting the density ρ equal to zero, and this has been verified by calculations in which the density term is retained. Finally the equations are solved subject to the boundary conditions

$$\begin{aligned} u(0,t) = u(h,t) &= 0, \\ -f(\theta)\frac{\partial\theta}{\partial z} - A_0\sin\theta\cos\theta + \eta\frac{\partial\theta}{\partial t} &= 0 \quad \text{at} \quad z = 0, \\ f(\theta)\frac{\partial\theta}{\partial z} - A_h\sin\theta\cos\theta + \eta\frac{\partial\theta}{\partial t} &= 0 \quad \text{at} \quad z = h. \end{aligned} \tag{6}$$

h denoting the cell thickness, η a surface viscosity, and A_0 and A_h positive constants.

NUMERICAL RESULTS

In this paper it is convenient to employ two different sets of initial conditions. The first of these is used when considering the full switch from uniform to bend states which involves the field being switched both on and off, and is

$$\theta(z,0) = \frac{\pi}{2} \text{ rad.}, \qquad u(z,0) = 0. \tag{7}$$

The second set represents only the second half of the above process where the field has already orientated the director to a near vertical state and is just about to be removed, and takes the form

$$\theta(z,0) = 1 \times 10^{-7} \text{ rad.}, \qquad u(z,0) = 0. \tag{8}$$

Since the flow plays such a large part in the switching process and is very unlikely to be identically zero across the cell at this stage, this final set of initial conditions is not ideal. However, it is the behaviour of the cell after the field has been switched off that is of most interest and the effect that different sets of material parameters have on this behaviour is more suitably compared from the same starting point.

For the results contained in this paper the material parameters for MBBA at 25°C. are considered (in cgs units)[4], that is

$$K_1 = 2.9 \times 10^{-7}, K_3 = 3.7 \times 10^{-7}, \alpha = 0.42, \eta = \alpha \times 10^{-6}, h = 2\mu m.$$

Results for other sets of parameters have been examined, however such results are qualitatively the same as those produced here. Further information regarding the specific problem being considered, such as the anchoring strength at both boundaries, will be given with the relevant figures.

In order to study the partial differential equations (2) and (3) with boundary conditions (6) and initial conditions (7) or (8) we use numerical techniques. This essentially involves employing the 'method of lines' to convert the system of partial differential equations, with finite differences, to a system of ordinary differential equations in the time direction[5].

It has been common practice in similar liquid crystal problems to neglect the inertial term ($\rho\frac{\partial u}{\partial t}$) in the equation for the balance of linear momentum (2). The first two figures in this paper compare solutions both with and without this inertial term. Figures 1 and 2 provide results obtained with the set of initial conditions (8). Figure 1 gives the values of the director angle θ at bottom, middle and top of the liquid crystal cell. The second figure gives two values of the velocity component u and, since the flow is always zero at the boundaries, these values are given at different points within the cell.

Although there is a difference in the two solutions it is possible to see from figure 1 that the transition to bend structure for the problem with the inertial term removed is only in the order of μs slower than the non-reduced problem. Not surprisingly, the results for the flow – given in figure 2 – show the solution neglecting the inertial term lagging behind, by a small amount, that of the other solution. This phenomenon is repeated in all the cases so far considered and, since it is a qualitative study of the switching process that is being attempted, the removal of the inertial term in further calculations can therefore be seen as a reasonable step. The main reason for wanting to neglect the left hand side of equation (2) is that it can take about 400 times as long to compute a solution with the inertial term as is does without it.

The following four figures give results for the given material parameters, starting this time with the initial conditions (7), and the surface anchoring strengths at the bottom and top plates given by $A_0 = 0.07$ and $A_h = 0.09$, respectively. The first two of these figures show the director and flow response to the applied electric field. The director – figure 3 – is seen to be attempting to align with the field by decreasing towards zero across the cell. The weak anchoring at the boundaries is shown by the fact that the director near both surfaces lags behind that in the bulk. For the field on, the flow, shown in figure 4, varies only slightly from an antisymmetric form, being approximately equal in magnitude and opposite in direction in the opposite halves of the cell.

When the electric field is removed the effect of the flow has a more pronounced influence on the liquid crystal cell. This is evident from figures 5 and 6, which give results for the field off. Simple elastic considerations suggest that the director orientation should simply return to its initial state ($\theta(z,t) = \frac{\pi}{2}, \forall z$) by way of a monotonic increase in θ across the cell. From knowledge of the backflow effect[6] we know that this simplistic relaxation will not take place. If the cell had strong anchoring at both boundaries then a shear flow would be generated – antisymmetric and opposite in sense to the flow with the field on. In a region around the centre of the cell a torque would be created which would oppose the elastic relaxation. Although this may take the director past the vertical ($\theta < 0$) in this region, the entire cell will eventually relax back to its initial state. Figures 5 and 6 make clear that this gradual relaxation back to a uniform state, parallel with the plates, is not happening in this weak anchoring problem.

From figure 5 it is possible to see that at the plate with the stronger anchoring, $z = h$, the director relaxes more swiftly back to its initial state. This induces flow in the negative direction near this plate which interferes with the relaxation in the rest of the cell. Figure 6 shows how this flow occurs throughout the cell as time progresses. The flow first appears significantly at the plate with faster relaxation and then diffuses across the cell before the elastic relaxation can take place at the slower ($z = 0$) surface. This produces a viscous torque in the lower part of the cell which opposes the intended elastic/surface elastic relaxation towards $\theta = \frac{\pi}{2}$ at the bottom plate. For this example it is apparent that the viscous torque is strong enough to overcome the elastic and surface elastic torques and the director orientation becomes negative near the lower plate. Once the director is negative the surface elasticity drives the surface orientation towards $-\frac{\pi}{2}$. This is shown in figure 5 where the director is shown to be tending towards a π-bend state with $\theta(0) = -\frac{\pi}{2}$ and $\theta(h) = \frac{\pi}{2}$. As discussed earlier, this bend state is unstable and must undergo a relaxation to the required π-twist state.

Finally in this section we turn our attention to considerations of how the time taken to switch between a uniform, planar state and a π-twist state may be decreased. Although we are only discussing the process where, via the application of an electric field, the uniform solution becomes a π-bend solution, the length of duration of this process surely gives some indication as to the length of time required for the full switch.

Figure 7 shows three different solutions corresponding to different sets of anchoring conditions. The solutions are in a form similar to that of figure 1, where this time the different line types separate the three different solutions. As with figure 1 the curves at the top of the figure represent the director angle at the top plate, those in the middle give θ-values in the centre of the cell and the remaining curves give the director orientation at $z = 0$. It is easy to see that, as the anchoring strength at both plates is increased, the time taken to proceed to a bend state decreases. This is not unexpected. Also, predictably, if the values of the anchoring strengths at each boundary are taken too close to each other then the cell will simply revert back to the uniform state when the electric field is removed.

CONCLUSIONS AND FURTHER WORK

Using continuum theory we have shown that the flow induced by the elastic/surface elastic relaxation at the faster boundary causes the elastic/surface elastic relaxation to be opposed at the weaker boundary, and leads to a π-bend state which must relax to a π-twist. Thus, we give theoretical proof that an asymmetrically anchored cell can switch from a uniform state to a twisted state with the application of an electric field. However, the switching times achieved so far appear to be slightly slower than those observed experimentally.

Attempts are currently being made to obtain a fuller understanding of the problem including the introduction of changes to the boundary conditions and the influence of twist effects.

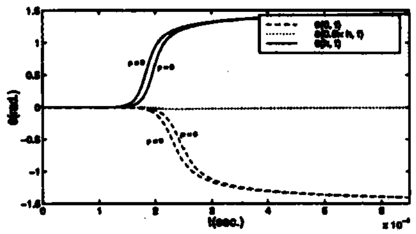

FIGURE 1 Director angle, $\theta(z, t)$, given at three points in a liquid crystal cell for two different values of density ρ.

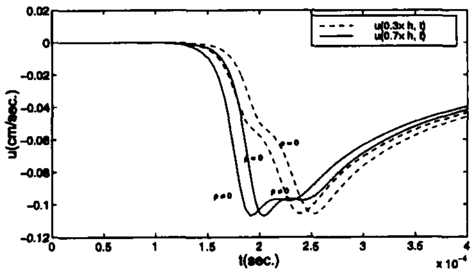

FIGURE 2 Flow, $u(z,t)$, given at two points within a liquid crystal cell for two different values of density ρ.

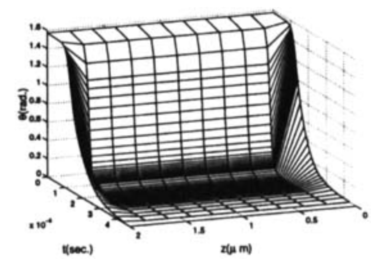

FIGURE 3 Director response to an applied electric field.

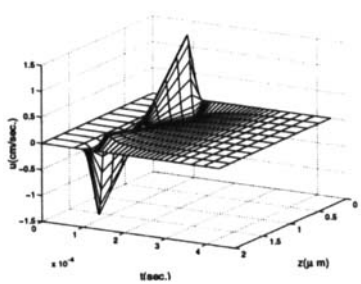

FIGURE 4 Flow induced by the electric field generated director reorientation.

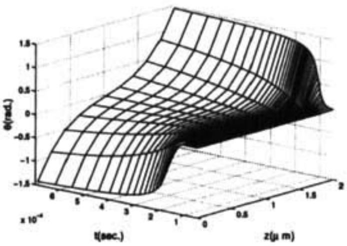

FIGURE 5 $\theta(z,t)$ after the field has been removed.

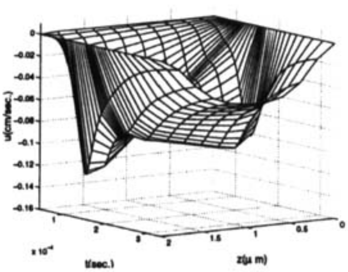

FIGURE 6 $u(z,t)$ after the field has been removed.

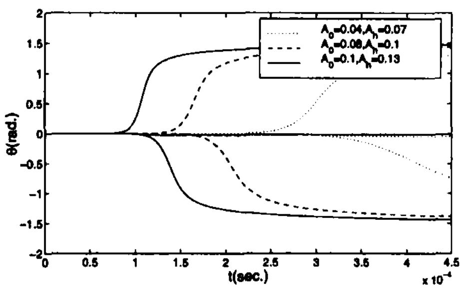

FIGURE 7 Three solutions for different sets of anchoring strengths given in terms of θ at three different points. Top curves: $\theta(z=h)$, middle curves: $\theta(z=\frac{h}{2})$ and bottom curves: $\theta(z=0)$.

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